An Unobserved Components Model of the Monetary Transmission Mechanism in a Closed Economy

Francis Vitek

Abstract

This paper develops and estimates an unobserved components model for purposes of monetary policy analysis in a closed economy. Cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Full information maximum likelihood estimation of this unobserved components model, conditional on prior information concerning the values of trend components, provides a quantitative description of the monetary transmission mechanism in a closed economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

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1. Introduction

In recent decades, the central banks of many economies have emphasized achieving inflation control objectives relative to achieving output stabilization objectives. Achieving low and stable inflation calls for accurate and precise indicators of inflationary pressure, together with an accurate and precise quantitative description of the monetary transmission mechanism.

A stylized qualitative description of the monetary transmission mechanism in a closed economy distinguishes among instruments, indicators, and targets. Given inflation control and output stabilization objectives, the central bank periodically adjusts a short term nominal interest rate in response to inflationary pressure. Provided that this response is sufficiently large, in the

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Affiliation: University of British Columbia

presence of short run nominal rigidities or imperfect information, an increase in the short term nominal interest rate causes an increase in the short term real interest rate, inducing intertemporal reductions in consumption and investment. In the presence of short run nominal rigidities or imperfect information, the resultant reduction in output is associated with a decline in inflation.

Despite the remarkable success of many central banks at achieving low and stable inflation in recent decades, the development of a mutually consistent set of accurate and precise indicators of inflationary pressure remains elusive. Theoretically prominent indicators of inflationary pressure such as the natural rate of interest are unobservable. As discussed in Woodford (2003), the natural rate of interest provides a measure of the neutral stance of monetary policy, with deviations of the real interest rate from the natural rate of interest generating inflationary pressure. Within the framework of an unobserved components model of selected elements of the monetary transmission mechanism in a closed economy, Laubach and Williams (2001, 2003) find that estimates of the natural rate of interest are relatively imprecise. Jointly estimating this and other indicators of inflationary pressure conditional on a larger information set may be expected to yield efficiency gains.

Definitions of indicators of inflationary pressure such as the natural rate of interest vary. Following Laubach and Williams (2001, 2003), we define the natural rate of interest as that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of shocks having temporary effects. In this long run equilibrium, there does not exist a cyclical stabilization role for monetary policy generated by nominal rigidities or imperfect information. In contrast, Woodford (2003) defines the natural rate of interest as that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of nominal rigidities. In this short run equilibrium, although there does not exist a cyclical stabilization role for monetary policy, the natural rate of interest varies in response to shocks having both temporary and permanent effects. Given an interest rate smoothing objective derived from a concern with financial market stability, it may be optimal for a central bank to adjust the short term nominal interest rate primarily in response to variation in the natural rate of interest caused by shocks having permanent effects.

Within the framework of a linear state space model, prior information concerning the values of unobserved state variables is often available in the form of deterministic or stochastic restrictions. Estimation of unobserved state variables with the filter due to Kalman (1960) does not exploit this prior information. Within the framework of an unobserved components model, prior information concerning the values of unobserved components is often available from alternative estimators. In the pursuit of efficiency gains in estimation of our unobserved components model of the monetary transmission mechanism in a closed economy, we extend the filter due to Kalman (1960) to incorporate prior information.

This paper develops and estimates an unobserved components model for purposes of monetary policy analysis in a closed economy. In an extension of the empirical framework developed by Laubach and Williams (2001, 2003), cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path. Although not derived from microeconomic foundations, this unobserved components model of the monetary transmission mechanism in a closed economy arguably provides a closer approximation to the data generating process than existing dynamic stochastic general equilibrium models. Full information maximum likelihood estimation of this unobserved components model, conditional on prior information concerning the values of trend components, provides a quantitative description of the monetary transmission mechanism in a closed economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

The organization of this paper is as follows. The next section develops an unobserved components model of the monetary transmission mechanism in a closed economy. In section three, unrestricted and restricted estimators of unobserved state variables are derived within the framework of a linear state space model. Estimation, inference and forecasting within the framework of a linear state space representation of our unobserved components model are the subjects of section four. Finally, section five offers conclusions and recommendations for further research.

2. The Unobserved Components Model

Consider a closed economy in which the central bank pursues inflation control and output stabilization objectives. Cyclical components are modeled as a multivariate linear rational expectations model of the monetary transmission mechanism, while trend components are modeled as random walks while ensuring the existence of a well defined balanced growth path.

2.1. Cyclical Components

The cyclical component of inflation depends on a linear combination of past and expected future cyclical components of inflation driven by the contemporaneous cyclical component of output according to Phillips curve

$$\hat{\pi}_t^P = \phi_{1,1}\hat{\pi}_{t-1}^P + \phi_{1,2}\mathrm{E}_t\hat{\pi}_{t+1}^P + \theta_{1,1}\ln\hat{Y}_t + \varepsilon_t^{\hat{P}},\ \varepsilon_t^{\hat{P}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{P}}^2), \tag{1}$$

where $\pi_t^P = \Delta \ln P_t$. The sensitivity of the cyclical component of inflation to changes in the cyclical component of output is increasing in $\theta_{1,1} > 0$.

The cyclical component of output follows a stationary second order autoregressive process driven by the past cyclical component of the real interest rate

$$\ln\hat{Y}_t = \phi_{2,1}\ln\hat{Y}_{t-1} + \phi_{2,2}\ln\hat{Y}_{t-2} + \theta_{2,1}\hat{r}_{t-1} + \varepsilon_t^{\hat{Y}},\ \varepsilon_t^{\hat{Y}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{Y}}^2), \tag{2}$$

where $r_t = i_t - \mathrm{E}_t\pi_{t+1}^P$. The sensitivity of the cyclical component of output to changes in the cyclical component of the real interest rate is decreasing in $\theta_{2,1} < 0$.

The cyclical component of consumption follows a stationary second order autoregressive process driven by the past cyclical component of the real interest rate:

$$\ln\hat{C}_t = \phi_{3,1}\ln\hat{C}_{t-1} + \phi_{3,2}\ln\hat{C}_{t-2} + \theta_{3,1}\hat{r}_{t-1} + \varepsilon_t^{\hat{C}},\ \varepsilon_t^{\hat{C}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{C}}^2). \tag{3}$$

The sensitivity of the cyclical component of consumption to changes in the cyclical component of the real interest rate is decreasing in $\theta_{3,1} < 0$.

The cyclical component of investment follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\ln\hat{I}_t = \phi_{4,1}\ln\hat{I}_{t-1} + \phi_{4,2}\ln\hat{I}_{t-2} + \theta_{4,1}\ln\hat{Y}_t + \varepsilon_t^{\hat{I}},\ \varepsilon_t^{\hat{I}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{I}}^2). \tag{4}$$

The sensitivity of the cyclical component of investment to changes in the cyclical component of output is increasing in $\theta_{4,1} > 0$.

The cyclical component of wage inflation depends on a linear combination of past and expected future cyclical components of wage inflation driven by the contemporaneous cyclical component of the unemployment rate according to wage Phillips curve

$$\begin{aligned}\hat{\pi}_t^W &= \phi_{5,1}\hat{\pi}_{t-1}^W + \phi_{5,2}\mathrm{E}_t\hat{\pi}_{t+1}^W + \theta_{5,1}\hat{u}_t \\ &\quad -\phi_{5,1}\theta_{5,2}\hat{\pi}_{t-1}^P + \theta_{5,2}\hat{\pi}_t^P - \phi_{5,2}\theta_{5,2}\mathrm{E}_t\hat{\pi}_{t+1}^P + \varepsilon_t^{\hat{W}},\ \varepsilon_t^{\hat{W}} \sim \text{iid } \mathcal{N}(0,\sigma_{\hat{W}}^2),\end{aligned} \tag{5}$$

where $\pi_t^W = \Delta \ln W_t$. The cyclical component of wage inflation also depends on past, contemporaneous, and expected future cyclical components of inflation. The sensitivity of the cyclical component of wage inflation to changes in the cyclical component of the unemployment rate is decreasing in $\theta_{5,1} < 0$, and to changes in the cyclical component of inflation is increasing in $0 < \theta_{5,2} < 1$.

The cyclical component of employment follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\ln \hat{L}_t = \phi_{6,1} \ln \hat{L}_{t-1} + \phi_{6,2} \ln \hat{L}_{t-2} + \theta_{6,1} \ln \hat{Y}_t + \varepsilon_t^{\hat{L}},\ \varepsilon_t^{\hat{L}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{L}}^2). \tag{6}$$

The sensitivity of the cyclical component of employment to changes in the cyclical component of output is increasing in $\theta_{6,1} > 0$.

The cyclical component of the unemployment rate follows a stationary second order autoregressive process driven by the contemporaneous cyclical component of output:

$$\hat{u}_t = \phi_{7,1} \hat{u}_{t-1} + \phi_{7,2} \hat{u}_{t-2} + \theta_{7,1} \ln \hat{Y}_t + \varepsilon_t^{\hat{u}},\ \varepsilon_t^{\hat{u}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{u}}^2). \tag{7}$$

The sensitivity of the cyclical component of the unemployment rate to changes in the cyclical component of output is decreasing in $\theta_{7,1} < 0$.

The cyclical component of the nominal interest rate follows a stationary first order autoregressive process driven by the contemporaneous cyclical components of inflation and output:

$$\hat{i}_t = \phi_{8,1} \hat{i}_{t-1} + \theta_{8,1} \hat{\pi}_t^P + \theta_{8,2} \ln \hat{Y}_t + \varepsilon_t^{\hat{i}},\ \varepsilon_t^{\hat{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\hat{i}}^2). \tag{8}$$

The sensitivity of the cyclical component of the nominal interest rate to changes in the cyclical component of inflation is increasing in $\theta_{8,1} > 0$, and to changes in the cyclical component of output is increasing in $\theta_{8,2} > 0$.

2.2. Trend Components

The trend components of output, consumption, and investment follow random walks with time varying drift $g_t + n_t$:

$$\ln \bar{Y}_t = g_t + n_t + \ln \bar{Y}_{t-1} + \varepsilon_t^{\bar{Y}},\ \varepsilon_t^{\bar{Y}} \sim \text{iid } \mathcal{N}(0, \sigma_{\bar{Y}}^2), \tag{9}$$

$$\ln \overline{C}_t = g_t + n_t + \ln \overline{C}_{t-1} + \varepsilon_t^{\overline{C}},\ \varepsilon_t^{\overline{C}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{C}}^2), \tag{10}$$

$$\ln \overline{I}_t = g_t + n_t + \ln \overline{I}_{t-1} + \varepsilon_t^{\overline{I}},\ \varepsilon_t^{\overline{I}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{I}}^2). \tag{11}$$

It follows that the trend components of the ratios of consumption and investment to output follow random walks without drift. This implies that along a balanced growth path, these great ratios are constant but state dependent.

The trend component of the price level follows a random walk with time varying drift π_t, the trend component of the nominal wage follows a random walk with time varying drift $\pi_t + g_t$, and the trend component of employment follows a random walk with time varying drift n_t:

$$\ln \overline{P}_t = \pi_t + \ln \overline{P}_{t-1} + \varepsilon_t^{\overline{P}},\ \varepsilon_t^{\overline{P}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{P}}^2), \tag{12}$$

$$\ln \overline{W}_t = \pi_t + g_t + \ln \overline{W}_{t-1} + \varepsilon_t^{\overline{W}},\ \varepsilon_t^{\overline{W}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{W}}^2), \tag{13}$$

$$\ln \overline{L}_t = n_t + \ln \overline{L}_{t-1} + \varepsilon_t^{\overline{L}},\ \varepsilon_t^{\overline{L}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{L}}^2). \tag{14}$$

It follows that the trend component of the income share of labour follows a random walk without drift. This implies that along a balanced growth path, the income share of labour is constant but state dependent.

The trend components of the unemployment rate and nominal interest rate follow random walks without drift:

$$\overline{u}_t = \overline{u}_{t-1} + \varepsilon_t^{\overline{u}},\ \varepsilon_t^{\overline{u}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{u}}^2), \tag{15}$$

$$\overline{i}_t = \overline{i}_{t-1} + \varepsilon_t^{\overline{i}},\ \varepsilon_t^{\overline{i}} \sim \text{iid } \mathcal{N}(0, \sigma_{\overline{i}}^2). \tag{16}$$

It follows that along a balanced growth path, the unemployment rate and nominal interest rate are constant but state dependent. The trend component of the real interest rate satisfies $\overline{r}_t = \overline{i}_t - \mathrm{E}_t \overline{\pi}_{t+1}^P$.

Long run nominal growth is driven by three common stochastic trends. Trend inflation, productivity growth, and population growth follow random walks without drift:

$$\pi_t = \pi_{t-1} + \varepsilon_t^{\pi},\ \varepsilon_t^{\pi} \sim \text{iid } \mathcal{N}(0, \sigma_{\pi}^2), \tag{17}$$

$$g_t = g_{t-1} + \varepsilon_t^g,\ \varepsilon_t^g \sim \text{iid } \mathcal{N}(0, \sigma_g^2), \tag{18}$$

$$n_t = n_{t-1} + \varepsilon_t^n,\ \varepsilon_t^n \sim \text{iid } \mathcal{N}(0, \sigma_n^2). \tag{19}$$

As an identifying restriction, all innovations are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

3. Estimation of Unobserved State Variables

Linear state space models consist of signal and state equations. The signal equation expresses a vector of observed nonpredetermined endogenous variables as a static deterministic or stochastic linear function of a vector of contemporaneous observed exogenous or predetermined endogenous variables, and a vector of contemporaneous unobserved state variables. The state equation expresses a vector of unobserved state variables as a dynamic deterministic or stochastic linear function of a vector of contemporaneous observed exogenous or predetermined endogenous variables, and a vector of lagged unobserved state variables.

Within the framework of a linear state space model, if the signal and state innovation vectors are multivariate normally distributed and contemporaneously uncorrelated, then conditional on the parameters associated with the signal and state equations, mean squared error optimal estimates of the unobserved state vector may be calculated with the filter due to Kalman (1960). If the signal and state innovation vectors are not multivariate normally distributed, then these state vector estimates retain minimum mean squared error status among the class of linear estimators. Estimation, inference and forecasting within the framework of a linear state space model is discussed in Hamilton (1994), Kim and Nelson (1999), and Durbin and Koopman (2001).

Within the framework of a linear state space model, prior information concerning the values of unobserved state variables is often available in the form of deterministic or stochastic restrictions. Estimation of unobserved state variables with the filter due to Kalman (1960) does not exploit this prior information. This section derives unrestricted and restricted estimators of unobserved state variables within the framework of a linear state space model. The former approach is standard, while the latter is a contribution of this paper. Exploiting prior information concerning the values of unobserved state variables may be expected to yield efficiency gains in estimation.

3.1. Unrestricted Estimation of Unobserved State Variables

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of N observed nonpredetermined endogenous variables, let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of M observed exogenous or predetermined endogenous variables, and let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of K unobserved state variables. Suppose that these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_t + \boldsymbol{A}_3\boldsymbol{\varepsilon}_{1,t}, \tag{20}$$

$$\boldsymbol{z}_t = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{2,t}, \tag{21}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0},\boldsymbol{\Sigma}_1)$ and $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0},\boldsymbol{\Sigma}_2)$. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Within the framework of this linear state space model, define $\boldsymbol{z}_{t|t-1} = \mathrm{E}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{P}_{t|t-1} = \mathrm{Var}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{y}_{t|t-1} = \mathrm{E}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$ and $\boldsymbol{Q}_{t|t-1} = \mathrm{Var}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, where $\mathcal{I}_{t-1} = \{\{\boldsymbol{y}_s\}_{s=1}^{t-1},\{\boldsymbol{x}_s\}_{s=1}^{t}\}$. Conditional on the parameters associated with the signal and state equations, these conditional means and variances satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1|t-1}, \tag{22}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{B}_2\boldsymbol{P}_{t-1|t-1}\boldsymbol{B}_2^{\mathsf{T}} + \boldsymbol{B}_3\boldsymbol{\Sigma}_2\boldsymbol{B}_3^{\mathsf{T}}, \tag{23}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_{t|t-1}, \tag{24}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{A}_2\boldsymbol{P}_{t|t-1}\boldsymbol{A}_2^{\mathsf{T}} + \boldsymbol{A}_3\boldsymbol{\Sigma}_1\boldsymbol{A}_3^{\mathsf{T}}. \tag{25}$$

These predicted estimates of the means and variances of the signal and state vectors are conditional on past information.

Given these predicted estimates, estimates of the state vector conditional on past and present information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|t}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \mathcal{I}_{t-1}) = \frac{f(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \mathcal{I}_{t-1}) f(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})}{f(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})}. \tag{26}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, $\boldsymbol{z}_{t|t}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1})^{\mathsf{T}} \boldsymbol{P}_{t|t-1}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1}) - (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{27}$$

subject to signal equation (20). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_t (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{28}$$

where $\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t-1}^{-1} - \boldsymbol{A}_2^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1} \boldsymbol{A}_2$ is positive definite. Define $\boldsymbol{P}_{t|t}$ as the mean squared error of $\boldsymbol{z}_{t|t}$, conditional on $\mathcal{I}_{t-1}$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_t \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1}. \tag{29}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|t}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \mathcal{I}_{t-1})$, and is therefore mean squared error optimal. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, recursive evaluation of equations (22), (23), (24), (25), (28) and (29) yields predicted and filtered estimates of the state vector.

Given these predicted and filtered estimates, estimates of the state vector conditional on past, present and future information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|T}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t) = \frac{f(\boldsymbol{z}_{t+1} \mid \boldsymbol{z}_t, \mathcal{I}_t) f(\boldsymbol{z}_t \mid \mathcal{I}_t)}{f(\boldsymbol{z}_{t+1} \mid \mathcal{I}_t)}. \tag{30}$$

Under the assumption of a multivariate normally distributed state innovation vector, $\boldsymbol{z}_{t|T}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t})^{\mathsf{T}} \boldsymbol{P}_{t|t}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t}) - (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t})^{\mathsf{T}} \boldsymbol{P}_{t+1|t}^{-1} (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t}), \tag{31}$$

subject to state equation (21). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t(\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{32}$$

where $\boldsymbol{J}_t = \boldsymbol{P}_{t|t}\boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t}^{-1} - \boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}\boldsymbol{B}_2$ is positive definite. Define $\boldsymbol{P}_{t|T}$ as the mean squared error of $\boldsymbol{z}_{t|T}$, conditional on $\mathcal{I}_t$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_t(\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t})\boldsymbol{J}_t^{\mathsf{T}}. \tag{33}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|T}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t)$, and is therefore mean squared error optimal. Given terminal conditions $\boldsymbol{z}_{T|T}$ and $\boldsymbol{P}_{T|T}$ obtained from the final evaluation of the prediction and updating equations, recursive evaluation of equations (32) and (33) yields smoothed estimates of the state vector.

3.2. Restricted Estimation of Unobserved State Variables

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of N observed nonpredetermined endogenous variables, let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of M observed exogenous or predetermined endogenous variables, and let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of K unobserved state variables. Suppose that these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{A}_1\boldsymbol{x}_t + \boldsymbol{A}_2\boldsymbol{z}_t + \boldsymbol{A}_3\boldsymbol{\varepsilon}_{1,t}, \tag{34}$$

$$\boldsymbol{z}_t = \boldsymbol{B}_1\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{z}_{t-1} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{2,t}, \tag{35}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_1)$ and $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_2)$. Let $\boldsymbol{w}_t$ denote a vector stochastic process consisting of J observed synthetic variables. Suppose that this vector stochastic process satisfies

$$\boldsymbol{w}_t = \boldsymbol{C}_1\boldsymbol{z}_t + \boldsymbol{C}_2\boldsymbol{\varepsilon}_{3,t}, \tag{36}$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_3)$. Conditional on given parameter values, this signal equation defines a set of deterministic or stochastic restrictions on linear combinations of unobserved state variables. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Within the framework of this linear state space model, define $\boldsymbol{z}_{t|t-1} = \mathrm{E}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{P}_{t|t-1} = \mathrm{Var}(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{y}_{t|t-1} = \mathrm{E}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{Q}_{t|t-1} = \mathrm{Var}(\boldsymbol{y}_t \mid \mathcal{I}_{t-1})$, $\boldsymbol{w}_{t|t-1} = \mathrm{E}(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})$ and $\boldsymbol{R}_{t|t-1} = \mathrm{Var}(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})$, where $\mathcal{I}_{t-1} = \{\{\boldsymbol{y}_s\}_{s=1}^{t-1}, \{\boldsymbol{w}_s\}_{s=1}^{t-1}, \{\boldsymbol{x}_s\}_{s=1}^{t}\}$. Conditional on the parameters associated with the signal and state equations, these conditional means and variances satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{B}_1 \boldsymbol{x}_t + \boldsymbol{B}_2 \boldsymbol{z}_{t-1|t-1}, \tag{37}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{B}_2 \boldsymbol{P}_{t-1|t-1} \boldsymbol{B}_2^{\mathsf{T}} + \boldsymbol{B}_3 \boldsymbol{\Sigma}_2 \boldsymbol{B}_3^{\mathsf{T}}, \tag{38}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{A}_1 \boldsymbol{x}_t + \boldsymbol{A}_2 \boldsymbol{z}_{t|t-1}, \tag{39}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\mathsf{T}} + \boldsymbol{A}_3 \boldsymbol{\Sigma}_1 \boldsymbol{A}_3^{\mathsf{T}}, \tag{40}$$

$$\boldsymbol{w}_{t|t-1} = \boldsymbol{C}_1 \boldsymbol{z}_{t|t-1}, \tag{41}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{C}_1 \boldsymbol{P}_{t|t-1} \boldsymbol{C}_1^{\mathsf{T}} + \boldsymbol{C}_2 \boldsymbol{\Sigma}_3 \boldsymbol{C}_2^{\mathsf{T}}. \tag{42}$$

These predicted estimates of the means and variances of the signal and state vectors are conditional on past information.

Given these predicted estimates, estimates of the state vector conditional on past and present information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|t}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1}) = \frac{f(\boldsymbol{y}_t \mid \boldsymbol{z}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1}) f(\boldsymbol{w}_t \mid \boldsymbol{z}_t, \mathcal{I}_{t-1}) f(\boldsymbol{z}_t \mid \mathcal{I}_{t-1})}{f(\boldsymbol{y}_t \mid \boldsymbol{w}_t, \mathcal{I}_{t-1}) f(\boldsymbol{w}_t \mid \mathcal{I}_{t-1})}. \tag{43}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, $\boldsymbol{z}_{t|t}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1})^{\top} \boldsymbol{P}_{t|t-1}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t-1}) \\ -(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\top} \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) - (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})^{\top} \boldsymbol{R}_{t|t-1}^{-1} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{44}$$

subject to signal equations (34) and (36). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_{\boldsymbol{y}_t} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) + \boldsymbol{K}_{\boldsymbol{w}_t} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{45}$$

where $\boldsymbol{K}_{\boldsymbol{y}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{A}_2^{\top} \boldsymbol{Q}_{t|t-1}^{-1}$ and $\boldsymbol{K}_{\boldsymbol{w}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{C}_1^{\top} \boldsymbol{R}_{t|t-1}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t-1}^{-1} - \boldsymbol{A}_2^{\top} \boldsymbol{Q}_{t|t-1}^{-1} \boldsymbol{A}_2 - \boldsymbol{C}_1^{\top} \boldsymbol{R}_{t|t-1}^{-1} \boldsymbol{C}_1$ is positive definite. Define $\boldsymbol{P}_{t|t}$ as the mean squared error of $\boldsymbol{z}_{t|t}$, conditional on $\mathcal{I}_{t-1}$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{y}_t} \boldsymbol{A}_2 \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{w}_t} \boldsymbol{C}_1 \boldsymbol{P}_{t|t-1}. \tag{46}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|t}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{y}_t, \boldsymbol{w}_t, \mathcal{I}_{t-1})$, and is therefore mean squared error optimal. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, recursive evaluation of equations (37), (38), (39), (40), (41), (42), (45) and (46) yields predicted and filtered estimates of the state vector.

Given these predicted and filtered estimates, estimates of the state vector conditional on past, present and future information may be derived with Bayesian updating. Define $\boldsymbol{z}_{t|T}$ as that argument which maximizes posterior distribution:

$$f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t) = \frac{f(\boldsymbol{z}_{t+1} \mid \boldsymbol{z}_t, \mathcal{I}_t) f(\boldsymbol{z}_t \mid \mathcal{I}_t)}{f(\boldsymbol{z}_{t+1} \mid \mathcal{I}_t)}. \tag{47}$$

Under the assumption of a multivariate normally distributed state innovation vector, $\boldsymbol{z}_{t|T}$ minimizes objective function

$$S(\boldsymbol{z}_t) = (\boldsymbol{z}_t - \boldsymbol{z}_{t|t})^{\top} \boldsymbol{P}_{t|t}^{-1} (\boldsymbol{z}_t - \boldsymbol{z}_{t|t}) - (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t})^{\top} \boldsymbol{P}_{t+1|t}^{-1} (\boldsymbol{z}_{t+1} - \boldsymbol{z}_{t+1|t}), \tag{48}$$

subject to state equation (35). The necessary first order condition associated with the implied unconstrained minimization problem yields

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t (\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{49}$$

where $\boldsymbol{J}_t = \boldsymbol{P}_{t|t}\boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}$. This necessary first order condition is sufficient if $\boldsymbol{P}_{t|t}^{-1} - \boldsymbol{B}_2^{\mathsf{T}}\boldsymbol{P}_{t+1|t}^{-1}\boldsymbol{B}_2$ is positive definite. Define $\boldsymbol{P}_{t|T}$ as the mean squared error of $\boldsymbol{z}_{t|T}$, conditional on $\mathcal{I}_t$. Within the framework of this linear state space model, this mean squared error matrix satisfies:

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_t(\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t})\boldsymbol{J}_t^{\mathsf{T}}. \tag{50}$$

Under our distributional assumptions, $\boldsymbol{z}_{t|T}$ equals the mean of posterior distribution $f(\boldsymbol{z}_t \mid \boldsymbol{z}_{t+1}, \mathcal{I}_t)$, and is therefore mean squared error optimal. Given terminal conditions $\boldsymbol{z}_{T|T}$ and $\boldsymbol{P}_{T|T}$ obtained from the final evaluation of the prediction and updating equations, recursive evaluation of equations (49) and (50) yields smoothed estimates of the state vector.

4. Estimation, Inference and Forecasting

Although unobserved components models feature prominently in the empirical macroeconomics literature, an unobserved components model of the monetary transmission mechanism has yet to be developed and estimated. Given that the monetary transmission mechanism is a cyclical phenomenon, it seems natural to model it within the framework of an unobserved components model.

4.1. Estimation

The traditional econometric interpretation of macroeconometric models regards them as representations of the joint probability distribution of the data. Adopting this traditional econometric interpretation, the parameters and trend components of our unobserved components model of the monetary transmission mechanism in a closed economy are jointly estimated by full information maximum likelihood, conditional on prior information concerning the values of trend components.

4.1.1. Estimation Methodology

Let $\boldsymbol{x}_t$ denote a vector stochastic process consisting of the levels of N nonpredetermined endogenous variables, of which M are observed. The cyclical components of this vector stochastic process satisfy third order stochastic linear difference equation

$$\boldsymbol{A}_0\hat{\boldsymbol{x}}_t = \boldsymbol{A}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{A}_2\hat{\boldsymbol{x}}_{t-2} + \boldsymbol{A}_3\mathrm{E}_t\hat{\boldsymbol{x}}_{t+1} + \boldsymbol{\varepsilon}_{1,t}, \tag{51}$$

where $\boldsymbol{\varepsilon}_{1,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_1)$. If there exists a unique stationary solution to this multivariate linear rational expectations model, then it may be expressed as:

$$\hat{\boldsymbol{x}}_t = \boldsymbol{B}_1\hat{\boldsymbol{x}}_{t-1} + \boldsymbol{B}_2\hat{\boldsymbol{x}}_{t-2} + \boldsymbol{B}_3\boldsymbol{\varepsilon}_{1,t}. \tag{52}$$

The trend components of vector stochastic process $\boldsymbol{x}_t$ satisfy first order stochastic linear difference equation

$$\boldsymbol{C}_0\overline{\boldsymbol{x}}_t = \boldsymbol{C}_1\boldsymbol{v}_t + \boldsymbol{C}_2\overline{\boldsymbol{x}}_{t-1} + \boldsymbol{\varepsilon}_{2,t}, \tag{53}$$

where $\boldsymbol{\varepsilon}_{2,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_2)$. Vector stochastic process $\boldsymbol{v}_t$ consists of the levels of L common stochastic trends, and satisfies first order stochastic linear difference equation

$$\boldsymbol{v}_t = \boldsymbol{v}_{t-1} + \boldsymbol{\varepsilon}_{3,t}, \tag{54}$$

where $\boldsymbol{\varepsilon}_{3,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_3)$. Cyclical and trend components are additively separable, which implies that $\boldsymbol{x}_t = \hat{\boldsymbol{x}}_t + \overline{\boldsymbol{x}}_t$.

Let $\boldsymbol{y}_t$ denote a vector stochastic process consisting of the levels of M observed nonpredetermined endogenous variables. Also, let $\boldsymbol{z}_t$ denote a vector stochastic process consisting of the contemporaneous levels of $N-M$ unobserved nonpredetermined endogenous variables, the contemporaneous and lagged cyclical components of N nonpredetermined endogenous variables, the contemporaneous trend components of N nonpredetermined endogenous variables, and the levels of L common stochastic trends. Given unique stationary solution (52), these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{F}_1\boldsymbol{z}_t, \tag{55}$$

$$\boldsymbol{z}_t = \boldsymbol{G}_1\boldsymbol{z}_{t-1} + \boldsymbol{G}_2\boldsymbol{\varepsilon}_{4,t}, \tag{56}$$

where $\boldsymbol{\varepsilon}_{4,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_4)$. Let $\boldsymbol{w}_t$ denote a vector stochastic process consisting of preliminary estimates of the trend components of M observed nonpredetermined endogenous variables. Suppose that this vector stochastic process satisfies

$$\boldsymbol{w}_t = \boldsymbol{H}_1\boldsymbol{z}_t + \boldsymbol{\varepsilon}_{5,t}, \tag{57}$$

where $\boldsymbol{\varepsilon}_{5,t} \sim \text{iid } \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_5)$. Conditional on given parameter values, this signal equation defines a set of deterministic or stochastic restrictions on selected unobserved state variables. The signal and state innovation vectors are assumed to be contemporaneously uncorrelated, which combined with our distributional assumptions implies independence.

Conditional on the parameters associated with these signal and state equations, estimates of unobserved state vector $\boldsymbol{z}_t$ and its mean squared error matrix $\boldsymbol{P}_t$ may be calculated with the filter derived previously. Given initial conditions $\boldsymbol{z}_{0|0}$ and $\boldsymbol{P}_{0|0}$, estimates conditional on information available at time $t-1$ satisfy prediction equations:

$$\boldsymbol{z}_{t|t-1} = \boldsymbol{G}_1\boldsymbol{z}_{t-1|t-1}, \tag{58}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{G}_1\boldsymbol{P}_{t-1|t-1}\boldsymbol{G}_1^{\mathsf{T}} + \boldsymbol{G}_2\boldsymbol{\Sigma}_4\boldsymbol{G}_2^{\mathsf{T}}, \tag{59}$$

$$\boldsymbol{y}_{t|t-1} = \boldsymbol{F}_1\boldsymbol{z}_{t|t-1}, \tag{60}$$

$$\boldsymbol{Q}_{t|t-1} = \boldsymbol{F}_1\boldsymbol{P}_{t|t-1}\boldsymbol{F}_1^{\mathsf{T}}, \tag{61}$$

$$\boldsymbol{w}_{t|t-1} = \boldsymbol{H}_1\boldsymbol{z}_{t|t-1}, \tag{62}$$

$$\boldsymbol{R}_{t|t-1} = \boldsymbol{H}_1\boldsymbol{P}_{t|t-1}\boldsymbol{H}_1^{\mathsf{T}} + \boldsymbol{\Sigma}_5. \tag{63}$$

Given these predictions, under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, estimates conditional on information available at time t satisfy updating equations

$$\boldsymbol{z}_{t|t} = \boldsymbol{z}_{t|t-1} + \boldsymbol{K}_{\boldsymbol{y}_t}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) + \boldsymbol{K}_{\boldsymbol{w}_t}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}), \tag{64}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{y}_t} \boldsymbol{F}_1 \boldsymbol{P}_{t|t-1} - \boldsymbol{K}_{\boldsymbol{w}_t} \boldsymbol{H}_1 \boldsymbol{P}_{t|t-1}, \tag{65}$$

where $\boldsymbol{K}_{\boldsymbol{y}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{F}_1^\top \boldsymbol{Q}_{t|t-1}^{-1}$ and $\boldsymbol{K}_{\boldsymbol{w}_t} = \boldsymbol{P}_{t|t-1} \boldsymbol{H}_1^\top \boldsymbol{R}_{t|t-1}^{-1}$. Given terminal conditions $\boldsymbol{z}_{T|T}$ and $\boldsymbol{P}_{T|T}$ obtained from the final evaluation of these prediction and updating equations, estimates conditional on information available at time T satisfy smoothing equations

$$\boldsymbol{z}_{t|T} = \boldsymbol{z}_{t|t} + \boldsymbol{J}_t (\boldsymbol{z}_{t+1|T} - \boldsymbol{z}_{t+1|t}), \tag{66}$$

$$\boldsymbol{P}_{t|T} = \boldsymbol{P}_{t|t} + \boldsymbol{J}_t (\boldsymbol{P}_{t+1|T} - \boldsymbol{P}_{t+1|t}) \boldsymbol{J}_t^\top, \tag{67}$$

where $\boldsymbol{J}_t = \boldsymbol{P}_{t|t} \boldsymbol{G}_1^\top \boldsymbol{P}_{t+1|t}^{-1}$. Under our distributional assumptions, these estimators of the unobserved state vector are mean squared error optimal.

Let $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^K$ denote a K dimensional vector containing the parameters associated with the signal and state equations of this linear state space model. The maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ of this parameter vector maximizes conditional loglikelihood function:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \ell_t(\boldsymbol{\theta}). \tag{68}$$

Under the assumption of multivariate normally distributed signal and state innovation vectors, together with conditionally contemporaneously uncorrelated signal vectors, the contributions to this conditional loglikelihood function satisfy $\ell_t(\boldsymbol{\theta}) = \ell_{\boldsymbol{y}_t}(\boldsymbol{\theta}) + \ell_{\boldsymbol{w}_t}(\boldsymbol{\theta})$, where:

$$\ell_{\boldsymbol{y}_t}(\boldsymbol{\theta}) = -\frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{Q}_{t|t-1}| - \frac{1}{2}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^\top \boldsymbol{Q}_{t|t-1}^{-1} (\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}), \tag{69}$$

$$\ell_{\boldsymbol{w}_t}(\boldsymbol{\theta}) = -\frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{R}_{t|t-1}| - \frac{1}{2}(\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1})^\top \boldsymbol{R}_{t|t-1}^{-1} (\boldsymbol{w}_t - \boldsymbol{w}_{t|t-1}). \tag{70}$$

Under regularity conditions stated in Watson (1989), maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ is consistent and asymptotically normal,

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} \mathcal{N}(\mathbf{0}, \boldsymbol{\mathcal{A}}_0^{-1} \boldsymbol{\mathcal{B}}_0 \boldsymbol{\mathcal{A}}_0^{-1}), \tag{71}$$

where $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$ denotes the true parameter vector. Following Engle and Watson (1981), consistent estimators of $\boldsymbol{\mathcal{A}}_0$ and $\boldsymbol{\mathcal{B}}_0$ are given by

$$\hat{\mathcal{A}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T), \tag{72}$$

$$\hat{\mathcal{B}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}, \tag{73}$$

where $\boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T) = \boldsymbol{a}_{y_t}(\hat{\boldsymbol{\theta}}_T) + \boldsymbol{a}_{w_t}(\hat{\boldsymbol{\theta}}_T)$ and $\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T) = \boldsymbol{b}_{y_t}(\hat{\boldsymbol{\theta}}_T) + \boldsymbol{b}_{w_t}(\hat{\boldsymbol{\theta}}_T)$. Under our distributional assumptions,

$$\boldsymbol{a}_{y_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1}^{\mathsf{T}}\boldsymbol{Q}_{t|t-1}^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1} + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}^{\mathsf{T}}(\boldsymbol{Q}_{t|t-1}^{-1}\otimes\boldsymbol{Q}_{t|t-1}^{-1})\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}, \tag{74}$$

$$\boldsymbol{a}_{w_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{w}_{t|t-1}^{\mathsf{T}}\boldsymbol{R}_{t|t-1}^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{w}_{t|t-1} + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{R}_{t|t-1}^{\mathsf{T}}(\boldsymbol{R}_{t|t-1}^{-1}\otimes\boldsymbol{R}_{t|t-1}^{-1})\nabla_{\boldsymbol{\theta}}\boldsymbol{R}_{t|t-1}, \tag{75}$$

$\boldsymbol{b}_{y_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\ell_{y_t}(\hat{\boldsymbol{\theta}}_T)$ and $\boldsymbol{b}_{w_t}(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\ell_{w_t}(\hat{\boldsymbol{\theta}}_T)$. If the signal innovation vectors are multivariate normally distributed, then the conditional information matrix equality holds, and $\mathcal{A}_0 = \mathcal{B}_0$.

4.1.2. Estimation Results

Our unobserved components model of the monetary transmission mechanism in a closed economy is estimated by full information maximum likelihood, conditional on prior information concerning the values of trend components. The data set consists of the levels of eight observed nonpredetermined endogenous variables for the United States described in Appendix A. The conditional loglikelihood function is maximized numerically using a modified steepest ascent algorithm. Estimation results pertaining to the period 1965Q1 through 2005Q2 appear in Appendix B, with robust t ratios reported in parentheses. The necessary and sufficient condition for the existence of a unique stationary rational expectations equilibrium due to Blanchard and Kahn (1980) is satisfied in a neighbourhood around the full information maximum likelihood estimate, while the analytical Hessian is nonsingular at the full information maximum likelihood estimate, suggesting that the linear state space representation of this unobserved components model is locally identified.

Prior information concerning the values of trend components is generated by fitting fourth order deterministic polynomial functions to the levels of observed nonpredetermined endogenous variables by ordinary least squares. Stochastic restrictions on the trend components of observed nonpredetermined endogenous variables are derived from the fitted values associated with these ordinary least squares regressions, with innovation variances set proportional to estimated prediction variances assuming known parameters. All stochastic restrictions are independent, represented by a diagonal covariance matrix, and are harmonized, represented by a common factor of proportionality. Reflecting little confidence in these preliminary trend component estimates, this common factor of proportionality is set equal to ten.

The signs of all parameter estimates are consistent with our priors, while most are statistically significant at conventional levels. Estimates of the variances of innovations associated with both cyclical and trend components are often statistically significant at conventional levels, suggesting that the levels of the observed nonpredetermined endogenous variables under consideration are subject to shocks having both temporary and permanent effects.

Predicted, filtered and smoothed estimates of the cyclical and trend components of observed nonpredetermined endogenous variables are plotted together with confidence intervals in Appendix B. These confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. The predicted estimates are conditional on past information, the filtered estimates are conditional on past and present information, and the smoothed estimates are conditional on past, present and future information. Visual inspection reveals close agreement with the conventional dating of business cycle expansions and recessions.

In order to examine whether our unobserved components model of the monetary transmission mechanism in a closed economy is dynamically complete in mean and variance, we subject the levels and squares of the predicted standardized residuals to the autocorrelation test of Ljung and Box (1978). We also examine whether there exist significant departures from conditional normality with the test of Jarque and Bera (1980). The predicted standardized residual vector $\boldsymbol{\zeta}_{t|t-1}$ is related to the predicted ordinary residual vector $\boldsymbol{\xi}_{t|t-1}$ by $\boldsymbol{\zeta}_{t|t-1} = \boldsymbol{Q}_{t|t-1}^{-1/2}\boldsymbol{\xi}_{t|t-1}$, where $\boldsymbol{\xi}_{t|t-1} = \boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}$. The inverse square root of predicted conditional covariance matrix $\boldsymbol{Q}_{t|t-1}$ is calculated with a spectral decomposition as $\boldsymbol{Q}_{t|t-1}^{-1/2} = \boldsymbol{X}_{t|t-1}\boldsymbol{\Lambda}_{t|t-1}^{-1/2}\boldsymbol{X}_{t|t-1}^{\top}$, where $\boldsymbol{X}_{t|t-1}$ denotes a square matrix containing distinct orthonormal eigenvectors, while $\boldsymbol{\Lambda}_{t|t-1}$ denotes a diagonal matrix containing the corresponding positive eigenvalues.

We find moderate evidence of autocorrelation in the predicted standardized residuals, suggesting that the conditional mean function is dynamically incomplete. Furthermore, we find strong evidence of autoregressive conditional heteroskedasticity in the predicted standardized residuals, suggesting that the conditional variance function is dynamically incomplete. Finally, we find strong evidence of departures from normality in the predicted standardized residuals, in part attributable to the existence of excess kurtosis. These residual diagnostic test results suggest that our full information maximum likelihood estimation results are consistent and asymptotically normal, but are asymptotically inefficient.

4.2. Inference

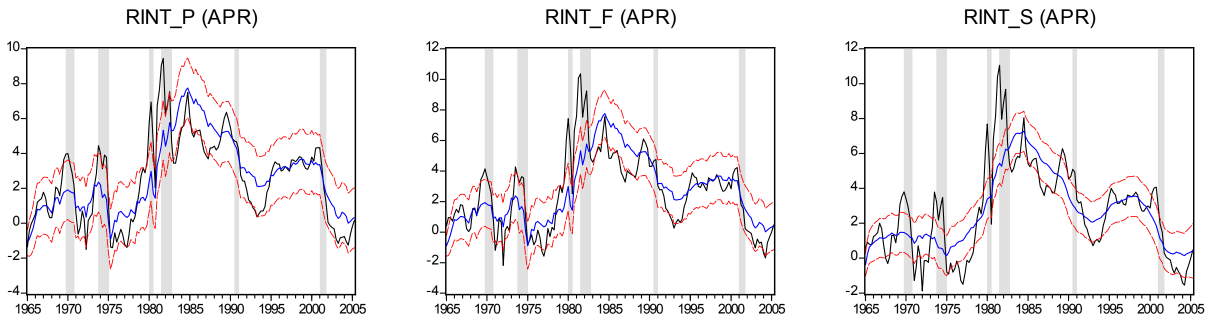
Achieving low and stable inflation calls for accurate and precise indicators of inflationary pressure, together with an accurate and precise quantitative description of the monetary transmission mechanism. Our unobserved components model of the monetary transmission mechanism in a closed economy addresses both of these challenges within a unified empirical framework.

4.2.1. Quantifying Inflationary Pressure

Theoretically prominent indicators of inflationary pressure such as the natural rate of interest are unobservable. As discussed in Woodford (2003), the natural rate of interest provides a measure of the neutral stance of monetary policy, with deviations of the real interest rate from the natural rate of interest generating inflationary pressure.

Predicted, filtered and smoothed estimates of the natural rate of interest are plotted together with confidence intervals versus corresponding estimates of the real interest rate in Figure 1. This concept of the natural rate of interest represents that short term real interest rate consistent with achieving inflation control and output stabilization objectives in the absence of shocks having temporary effects. Visual inspection reveals that our estimates of the natural rate of interest exhibit persistent low frequency variation and are relatively precise. Deviations of the estimated real interest rate from the estimated natural rate of interest are in close agreement with the conventional dating of business cycle expansions and recessions.

Figure 1. Predicted, filtered and smoothed estimates of the natural rate of interest



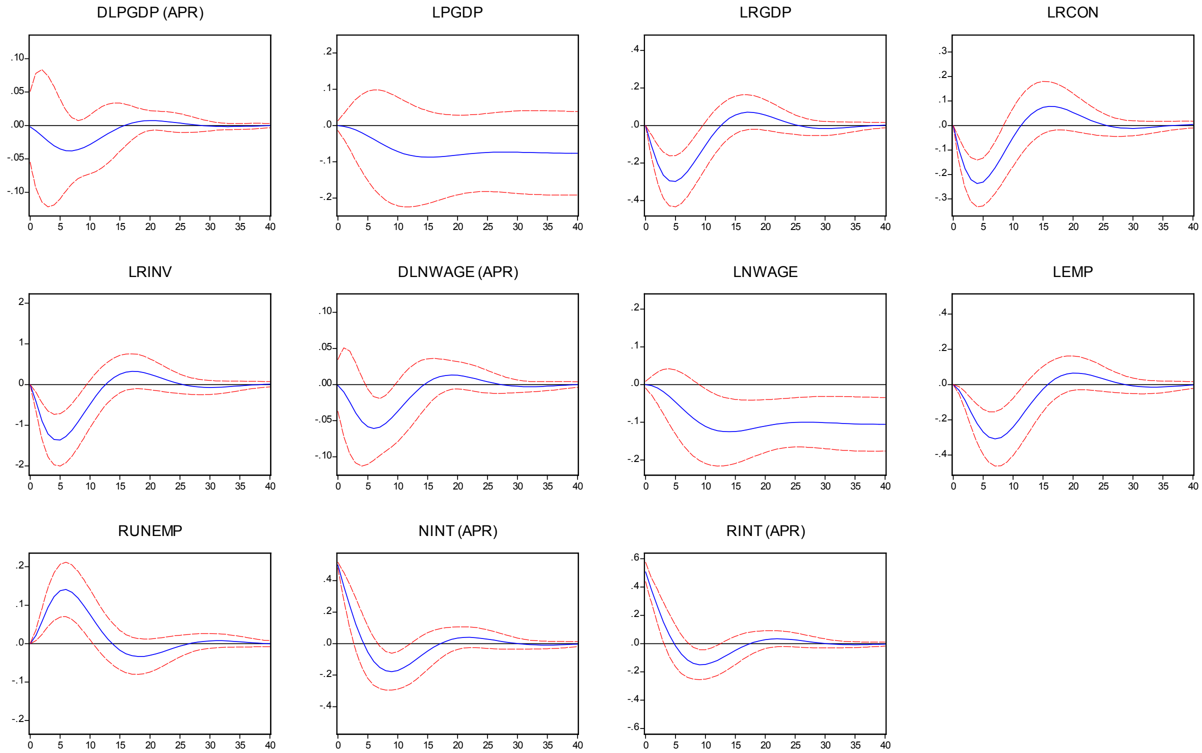
Note: Estimated levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

4.2.2. Quantifying the Monetary Transmission Mechanism

The monetary transmission mechanism describes the dynamic effects of unsystematic variation in the instrument of monetary policy on indicators and targets. In a closed economy, the monetary transmission mechanism features an interest rate channel. Estimated impulse responses to a monetary policy shock are plotted together with confidence intervals in Figure 2, providing a quantitative description of the monetary transmission mechanism in a closed economy.

In response to a monetary policy shock, the nominal and real interest rates exhibit immediate increases followed by gradual declines. These real interest rate dynamics induce persistent hump shaped reductions in output, consumption and investment, together with a persistent hump shaped decline in inflation, with peak effects realized after one to two years. These output dynamics are associated with a persistent hump shaped reduction in employment, together with a persistent hump shaped increase in the unemployment rate, inducing a persistent hump shaped decline in wage inflation, with peak effects realized after one to two years. These results are qualitatively consistent with those of structural vector autoregressive analyses of the monetary transmission mechanism in closed economies such as Sims and Zha (1995), Gordon and Leeper (1994), Leeper, Sims and Zha (1996), and Christiano, Eichenbaum and Evans (1998, 2005).

Figure 2. Estimated impulse responses to a monetary policy shock



Note: Estimated impulse responses to a 50 basis point monetary policy shock are depicted. Symmetric 95% confidence intervals are calculated with the delta method.

4.3. Forecasting

While it is desirable that forecasts be unbiased and efficient, the practical value of any forecasting model depends on its relative predictive accuracy. As a benchmark against which to evaluate the predictive accuracy of our unobserved components model of the monetary transmission mechanism in a closed economy, we consider the autoregressive integrated moving average or ARIMA class of models. In particular, we consider ARIMA models for scalar stochastic process $\{y_t\}_{t=1}^{T}$ of the form

$$\Delta^d y_t = \mu + \sum_{i=1}^{p} \phi_i \Delta^d y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \tag{76}$$

where $\varepsilon_t \sim \text{iid } \mathcal{N}(0,\sigma^2)$. Theoretical support for this univariate forecasting framework is provided by the decomposition theorem due to Wold (1938), which states that any covariance stationary purely linearly indeterministic scalar stochastic process has an infinite order moving average representation. As discussed in Clements and Hendry (1998), any infinite order moving average process can be approximated to any required degree of accuracy by an autoregressive moving average process, with the required autoregressive and moving average orders typically being relatively low.

The ARIMA models are estimated by maximum likelihood over the period 1965Q3 through 2005Q2. The autoregressive, ordinary difference, and moving average orders are jointly selected to minimize the model selection criterion function proposed by Schwarz (1978). Those ARIMA model specifications deemed optimal are reported in Table 1.

Table 1. Optimal ARIMA model specifications

y_t	p	d	q
$\ln P_t$	2	0	1
$\ln Y_t$	4	1	2
$\ln C_t$	0	2	2
$\ln I_t$	0	2	1
$\ln W_t$	2	0	1
$\ln L_t$	1	2	1
u_t	2	0	0
i_t	1	2	2

Note: The autoregressive order p, ordinary difference order d, and moving average order q are jointly selected subject to upper bounds of four, two and two, respectively.

In the absence of a well defined mapping between forecast errors and their costs, relative predictive accuracy is generally assessed with mean squared prediction error based measures. As discussed in Clements and Hendry (1998), mean squared prediction error based measures are noninvariant to nonsingular, scale preserving linear transformations, even though linear models are. It follows that mean squared prediction error based comparisons may yield conflicting rankings across models, depending on the variable transformations examined.

To evaluate the dynamic out of sample forecasting performance of our unobserved components model of the monetary transmission mechanism in a closed economy, we retain forty quarters of observations to evaluate forecasts one through eight quarters ahead, generated conditional on parameters estimated using information available at the forecast origin. The models are compared on the basis of mean squared prediction errors in levels, ordinary differences, and seasonal differences. The unobserved components model is not recursively estimated as the forecast origin rolls forward due to the high computational cost of such a procedure, while the ARIMA models are. Presumably, recursively estimating the unobserved components model would increase its predictive accuracy.

Mean squared prediction error differentials are plotted together with confidence intervals accounting for contemporaneous and serial correlation of forecast errors in Appendix B. If these mean squared prediction error differentials are negative then the forecasting performance of the unobserved components model dominates that of the ARIMA models, while if positive then the unobserved components model is dominated by the ARIMA models in terms of predictive accuracy. The null hypothesis of equal squared prediction errors is rejected by the predictive accuracy test of Diebold and Mariano (1995) if and only if these confidence intervals exclude zero. The asymptotic variance of the average loss differential is estimated by a weighted sum of the autocovariances of the loss differential, employing the weighting function proposed by Newey and West (1987). Visual inspection reveals that these mean squared prediction error differentials are of variable sign, suggesting that the unobserved components model matches the ARIMA models in terms of predictive accuracy, in spite of a considerable informational disadvantage. However, these mean squared prediction error differentials are rarely statistically significant at conventional levels, indicating that considerable uncertainty surrounds these forecast accuracy comparisons.

Dynamic out of sample forecasts of levels, ordinary differences, and seasonal differences are plotted together with confidence intervals versus realized outcomes in Appendix B. These confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Visual inspection reveals that the realized outcomes generally lie within their associated confidence intervals, suggesting that forecast failure is absent. However, these confidence intervals are rather wide, indicating that considerable uncertainty surrounds the point forecasts.

5. Conclusion

This paper develops and estimates an unobserved components model of the monetary transmission mechanism in a closed economy for purposes of monetary policy analysis. This estimated unobserved components model provides a quantitative description of the monetary transmission mechanism in a closed economy, yields a mutually consistent set of indicators of inflationary pressure together with confidence intervals, and facilitates the generation of relatively accurate forecasts.

In an open economy, the monetary transmission mechanism features both interest rate and exchange rate channels, while a central bank having inflation control and output stabilization objectives must react to shocks originating both domestically and abroad. The extension of our unobserved components model of the monetary transmission mechanism in a closed economy to an open economy framework remains an objective for future research.

Acknowledgements

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Appendix A. Description of the Data Set

The data set consists of quarterly seasonally adjusted observations on several macroeconomic variables for the United States over the period 1964Q1 through 2005Q2. All aggregate prices and quantities are expenditure based. Employment is derived from observed nominal labour income and a nominal wage index, while the unemployment rate is quoted as a period average. The nominal interest rate is measured by the federal funds rate quoted as a period average. All data was extracted from the FRED database maintained by the Federal Reserve Bank of Saint Louis.

Appendix B. Tables and Figures

Table 2. Full information maximum likelihood estimation results

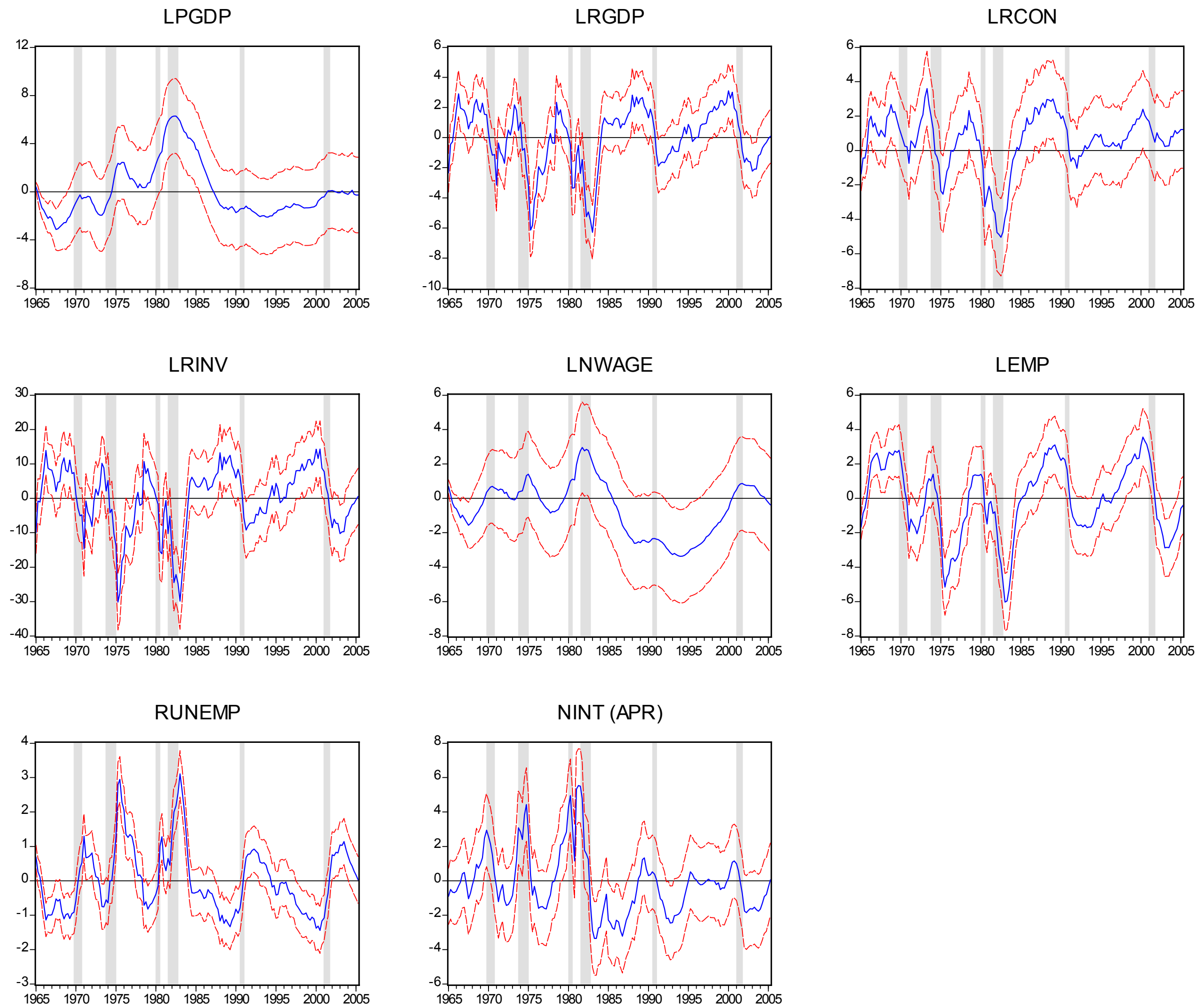
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{4,1}$	$\phi_{4,2}$
0.604	0.256	1.125	–0.176	1.013	–0.097	0.273	–0.130
(0.550)	(0.126)	(11.740)	(–1.883)	(8.868)	(–0.908)	(3.037)	(–1.919)
$\phi_{5,1}$	$\phi_{5,2}$	$\phi_{6,1}$	$\phi_{6,2}$	$\phi_{7,1}$	$\phi_{7,2}$	$\phi_{8,1}$	
0.034	0.006	0.941	–0.168	0.797	–0.200	0.781	
(0.133)	(0.003)	(6.322)	(–1.352)	(9.987)	(–3.337)	(11.761)	
$\theta_{1,1}$	$\theta_{2,1}$	$\theta_{3,1}$	$\theta_{4,1}$	$\theta_{5,1}$	$\theta_{5,2}$	$\theta_{6,1}$	$\theta_{7,1}$
0.006	–0.851	–0.792	3.866	–0.069	0.539	0.296	–0.200
(0.172)	(–3.474)	(–3.789)	(9.561)	(–0.480)	(5.865)	(7.051)	(–8.268)
$\theta_{8,1}$	$\theta_{8,2}$						
0.330	0.052						
(2.500)	(4.054)						
$\sigma^2_{\hat{P}}$	$\sigma^2_{\hat{Y}}$	$\sigma^2_{\hat{C}}$	$\sigma^2_{\hat{I}}$	$\sigma^2_{\hat{W}}$	$\sigma^2_{\hat{L}}$	$\sigma^2_{\hat{u}}$	$\sigma^2_{\hat{i}}$
0.023	0.431	0.244	3.023	0.031	0.089	0.013	0.038
(0.251)	(5.453)	(6.123)	(4.518)	(1.949)	(2.075)	(1.930)	(3.383)
$\sigma^2_{\bar{P}}$	$\sigma^2_{\bar{Y}}$	$\sigma^2_{\bar{C}}$	$\sigma^2_{\bar{I}}$	$\sigma^2_{\bar{W}}$	$\sigma^2_{\bar{L}}$	$\sigma^2_{\bar{u}}$	$\sigma^2_{\bar{i}}$
0.050	0.079	0.079	1.988	0.053	0.125	0.017	0.007
(5.362)	(2.931)	(2.532)	(2.903)	(3.121)	(2.457)	(2.886)	(1.981)
σ^2_{π}	σ^2_{g}	σ^2_{n}					
1.28×10^{-3}	2.22×10^{-5}	1.90×10^{-5}					
(2.472)	(1.796)	(0.443)					

	$\hat{\zeta}^P_{t\mid t-1}$	$\hat{\zeta}^Y_{t\mid t-1}$	$\hat{\zeta}^C_{t\mid t-1}$	$\hat{\zeta}^I_{t\mid t-1}$	$\hat{\zeta}^W_{t\mid t-1}$	$\hat{\zeta}^L_{t\mid t-1}$	$\hat{\zeta}^u_{t\mid t-1}$	$\hat{\zeta}^i_{t\mid t-1}$
$Q(2)$	12.939***	12.527***	2.887	5.785*	22.637***	11.510***	3.017	6.870**
$Q(4)$	32.963***	14.504***	13.451***	10.061**	49.721***	18.637***	3.968	19.846***
$Q^2(2)$	69.862***	25.454***	19.334***	8.738**	15.586***	67.689***	32.142***	13.881***
$Q^2(4)$	100.638***	34.563***	42.573***	19.194***	30.930***	110.794***	52.811***	39.584***
Skewness	0.663***	0.337*	0.276	0.515***	0.050	−0.148	0.667***	1.799***
Kurtosis	3.861**	4.871***	4.280***	6.199***	5.276***	3.869**	4.828***	13.151***
JB	16.863***	26.700***	13.112***	76.217***	35.032***	5.689*	34.592***	782.850***

$\mathcal{L}(\hat{\boldsymbol{\theta}}_T) = -4468.430$

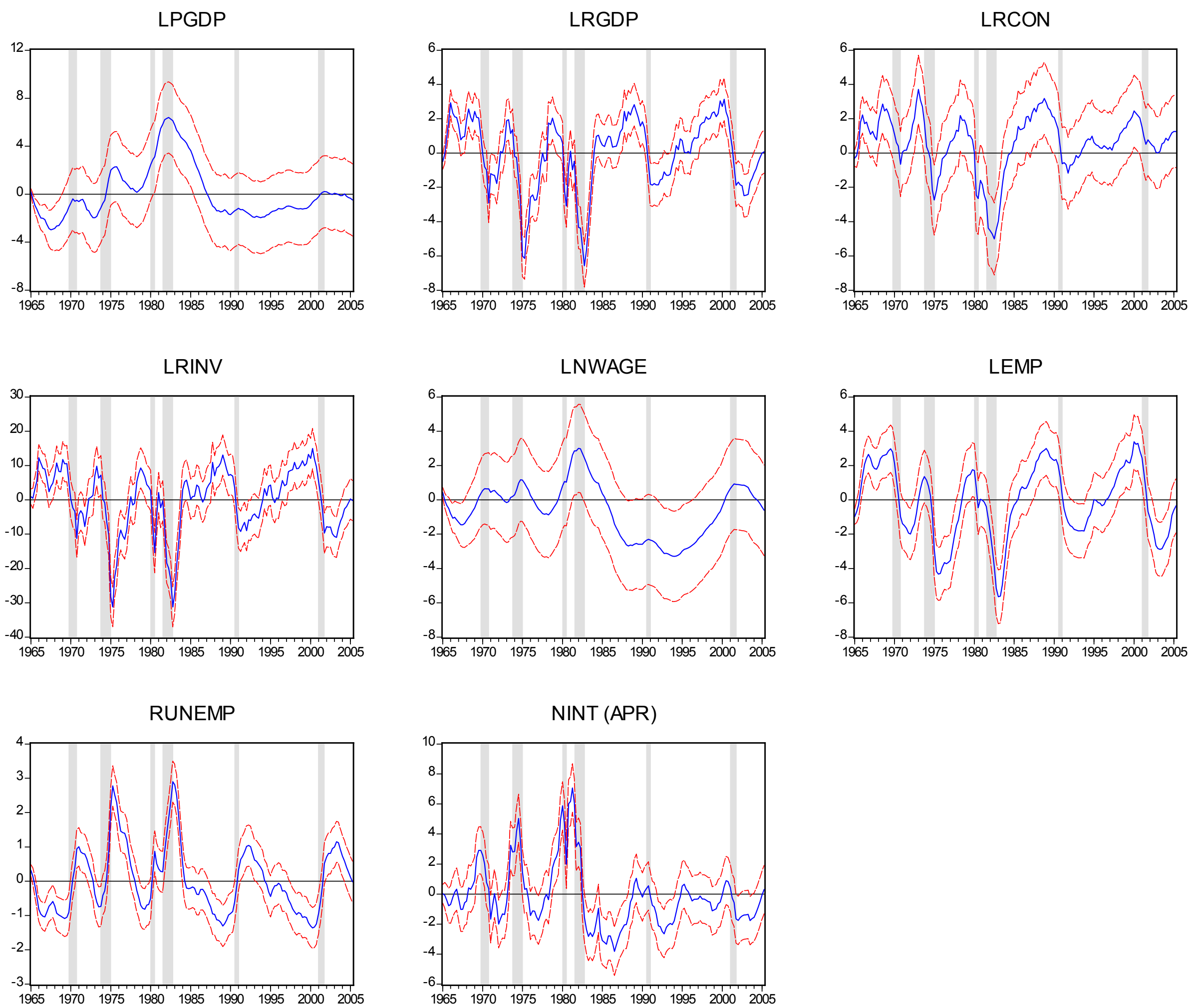
Note: Rejection of the null hypothesis at the 1%, 5% and 10% levels is indicated by ***, ** and *, respectively.

Figure 3. Predicted cyclical components of observed nonpredetermined endogenous variables



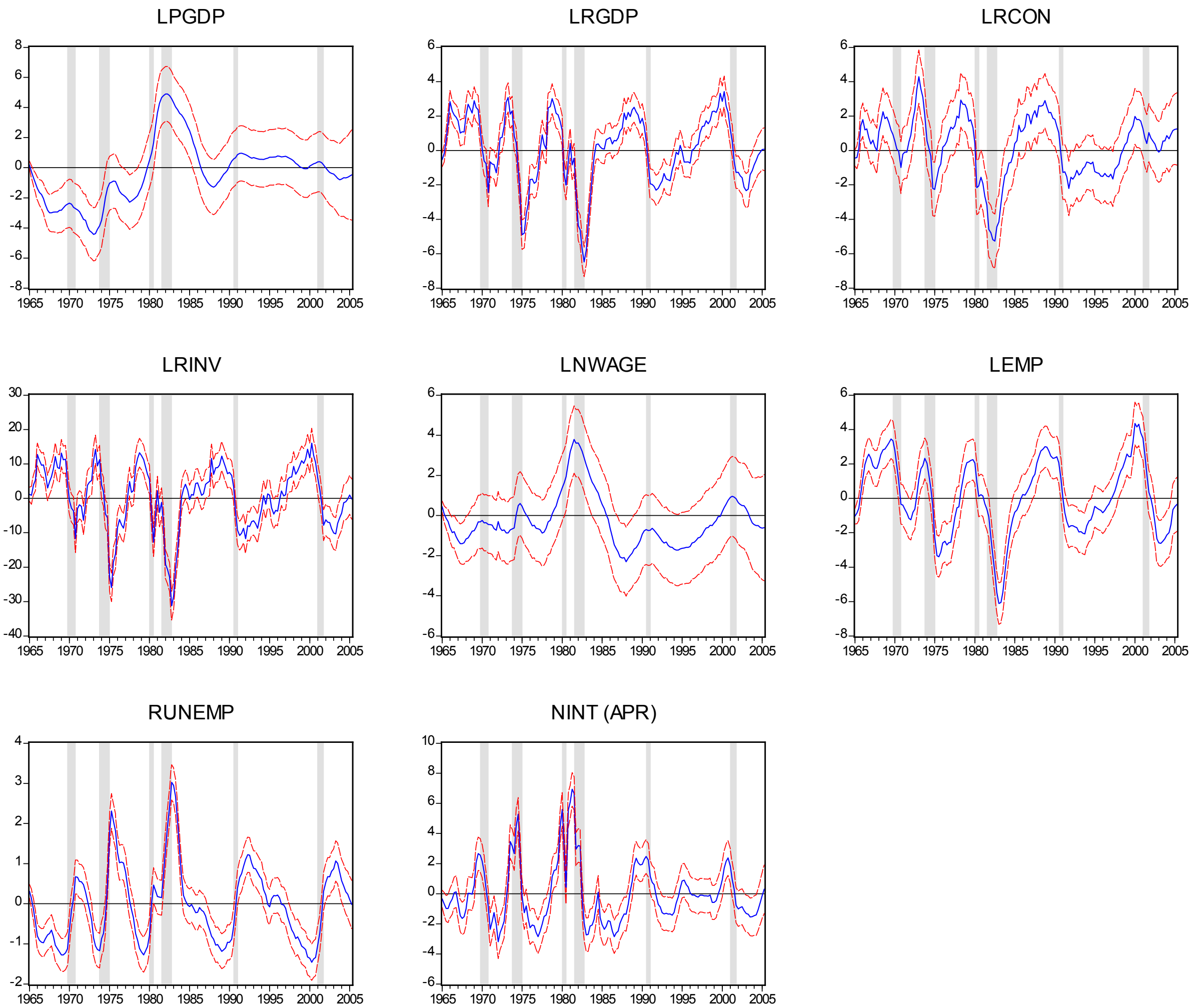
Note: Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 4. Filtered cyclical components of observed nonpredetermined endogenous variables



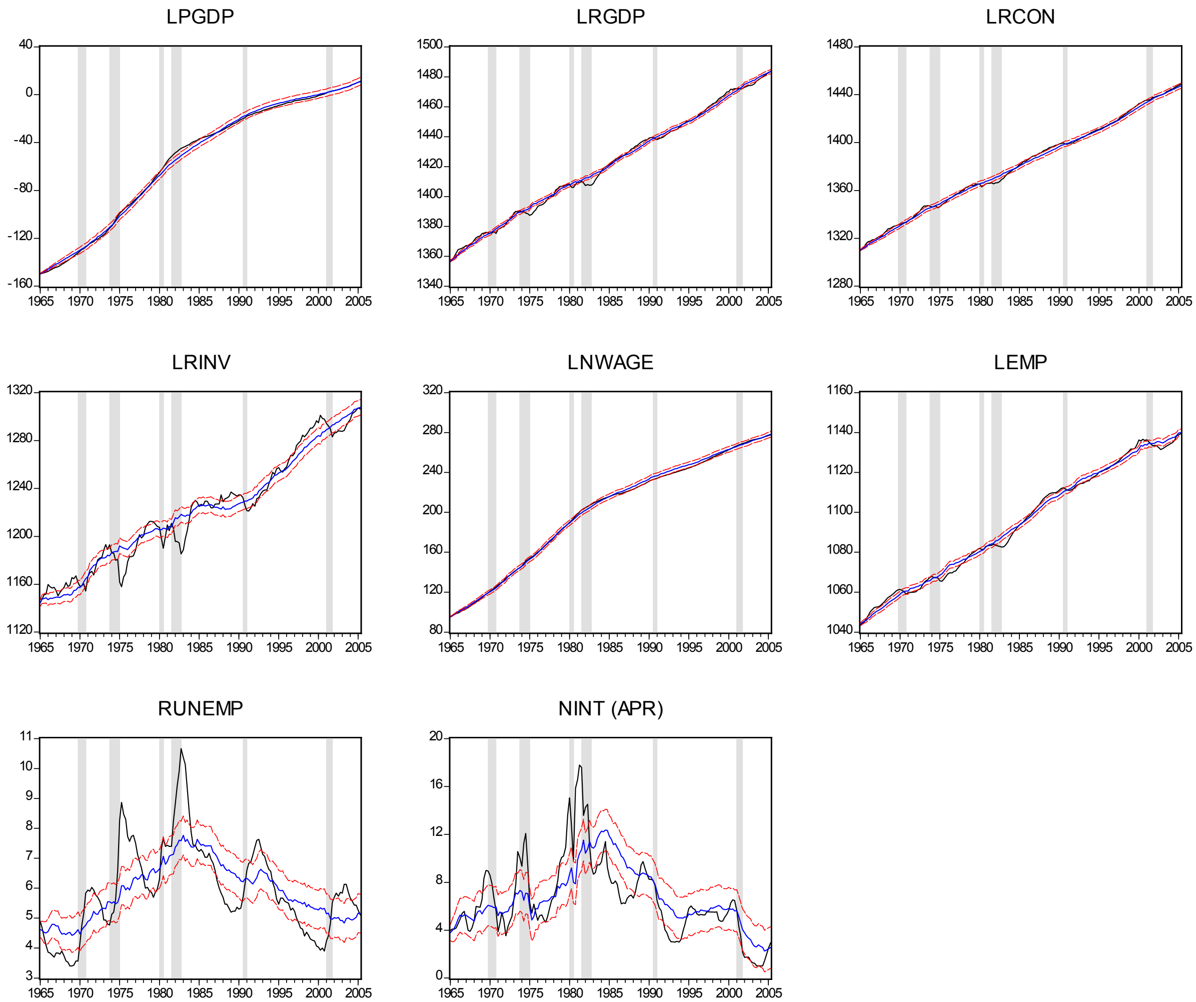
Note: Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 5. Smoothed cyclical components of observed nonpredetermined endogenous variables



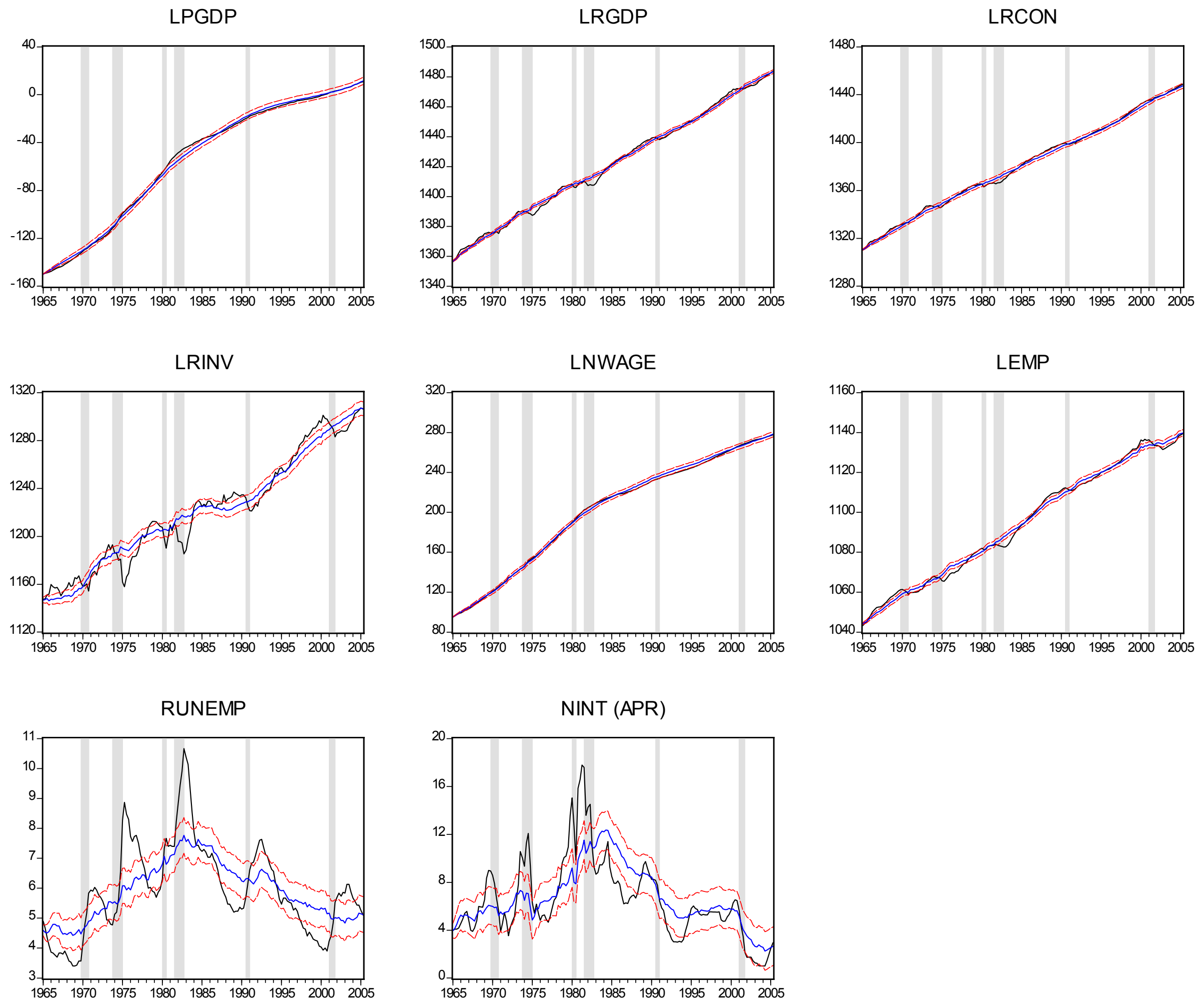
Note: Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 6. Predicted trend components of observed nonpredetermined endogenous variables



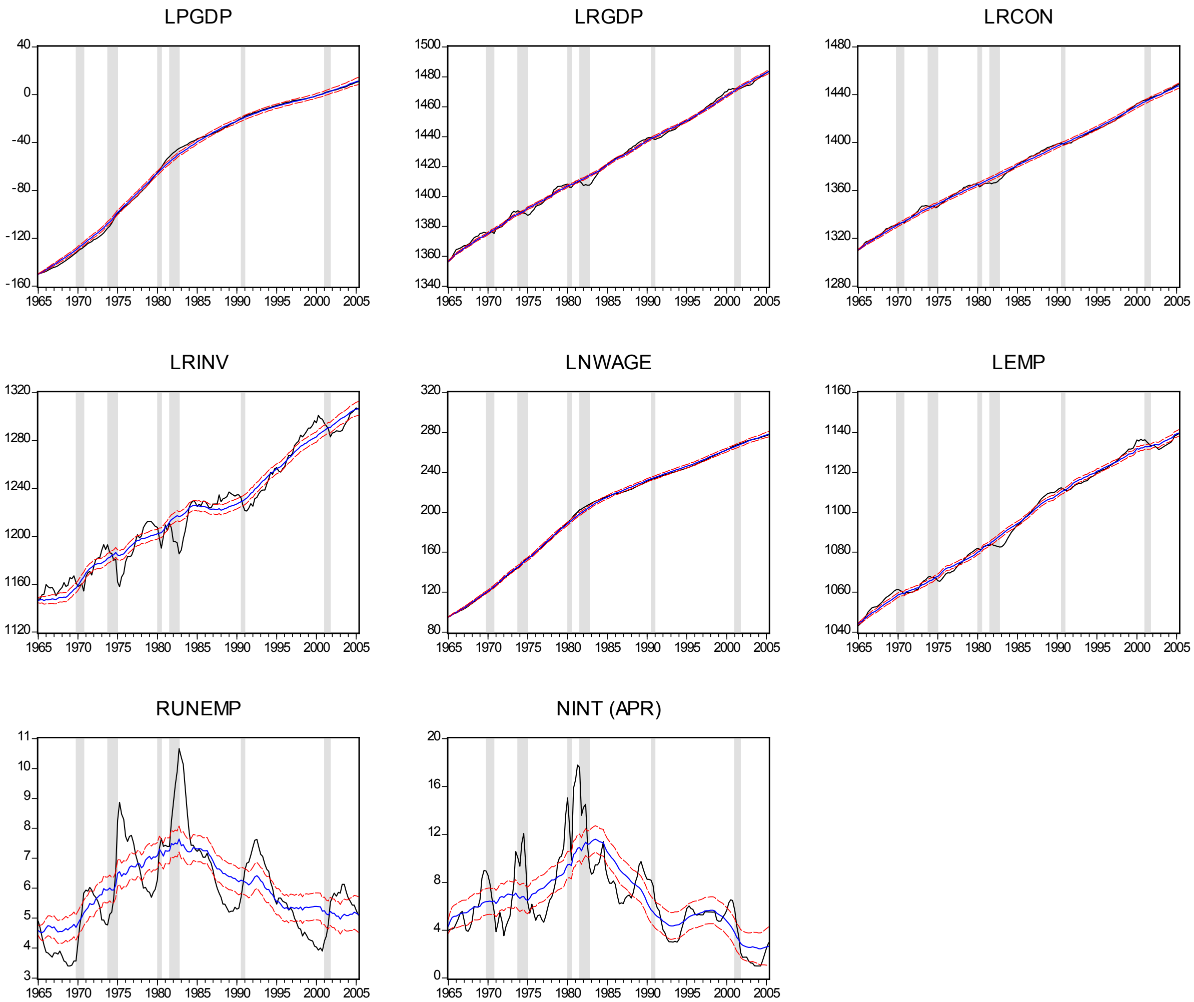
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 7. Filtered trend components of observed nonpredetermined endogenous variables



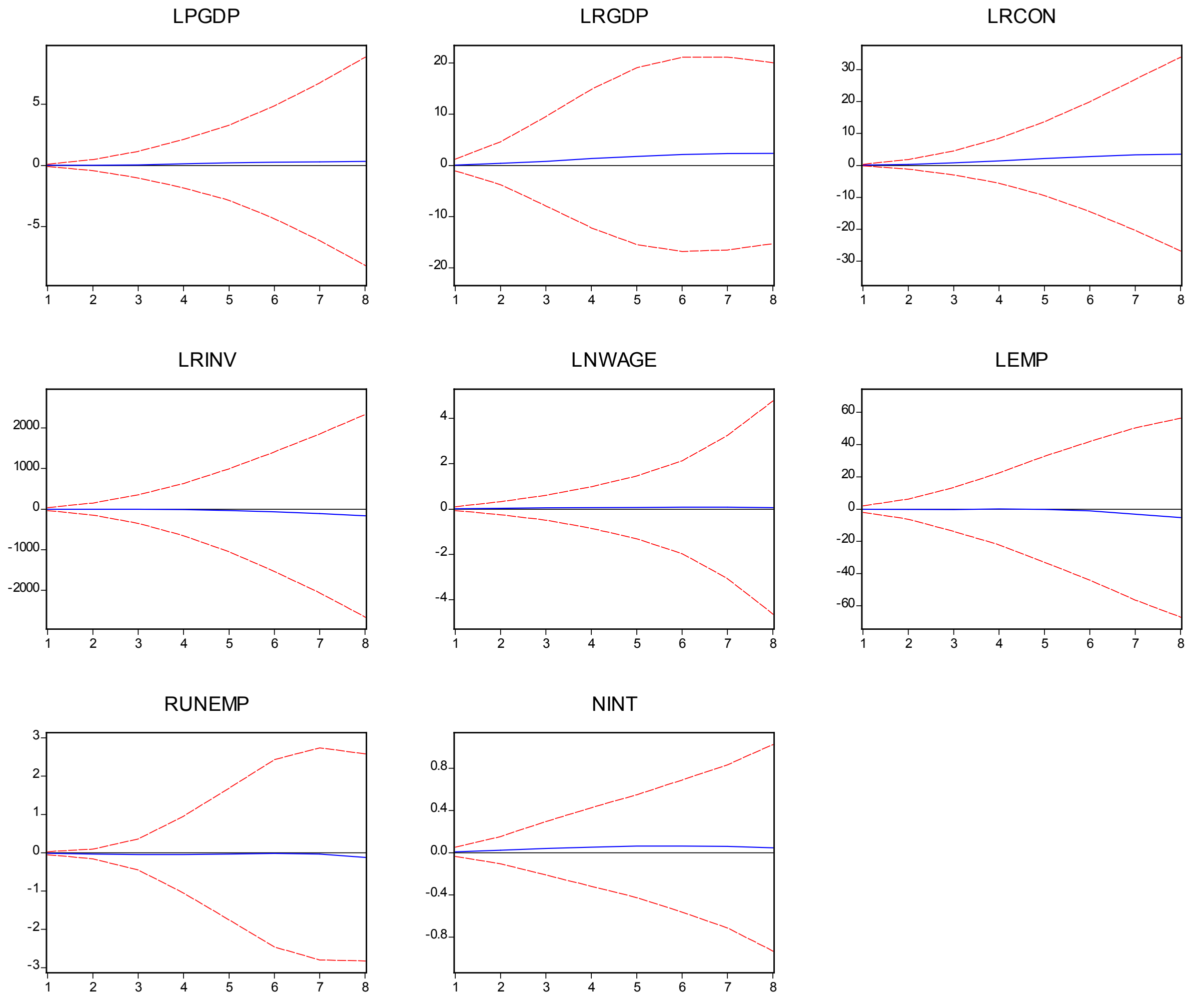
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 8. Smoothed trend components of observed nonpredetermined endogenous variables



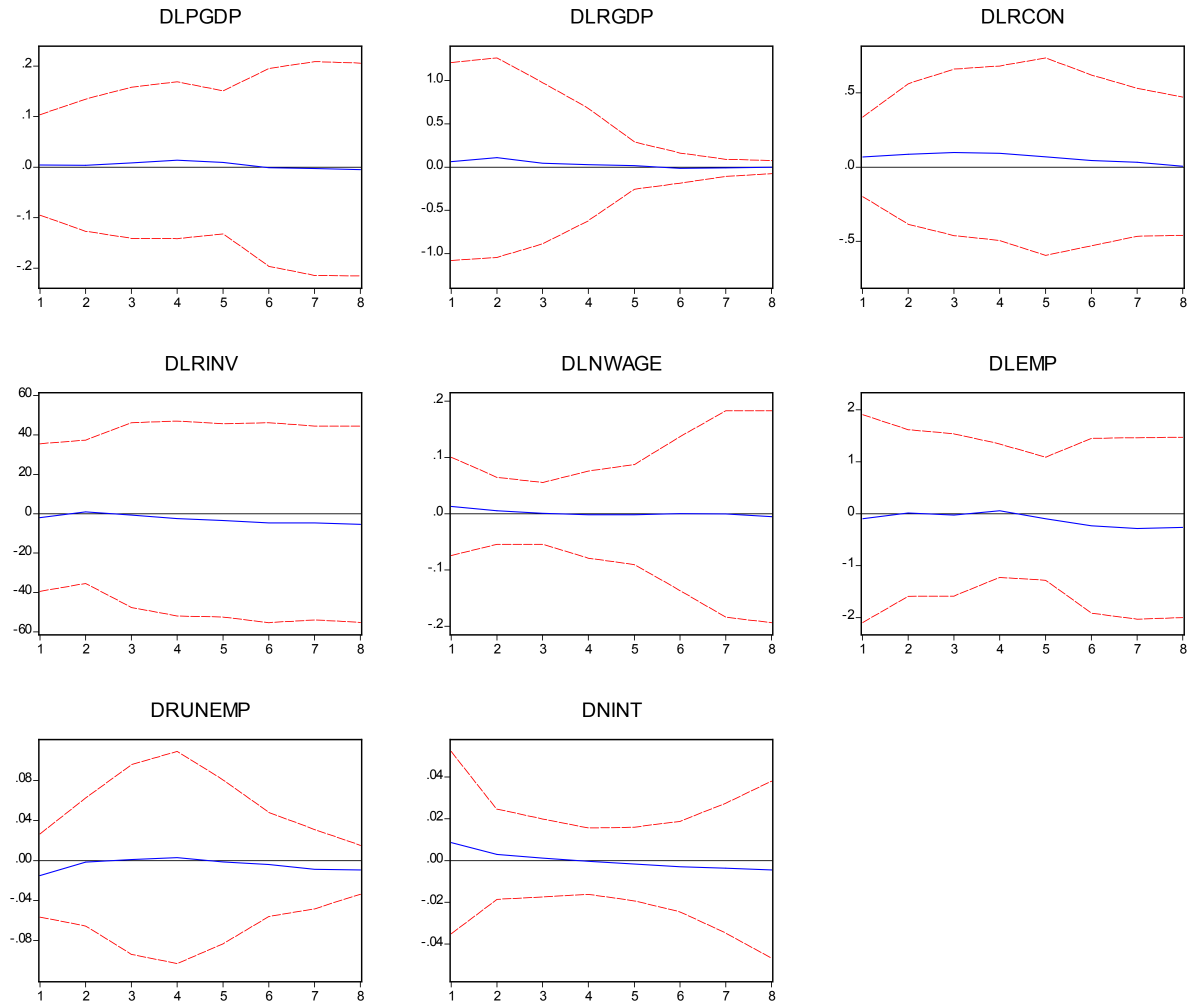
Note: Observed levels are represented by black lines, while blue lines depict estimated trend components. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters. Shaded regions indicate recessions as dated by the National Bureau of Economic Research reference cycle.

Figure 9. Mean squared prediction error differentials for levels



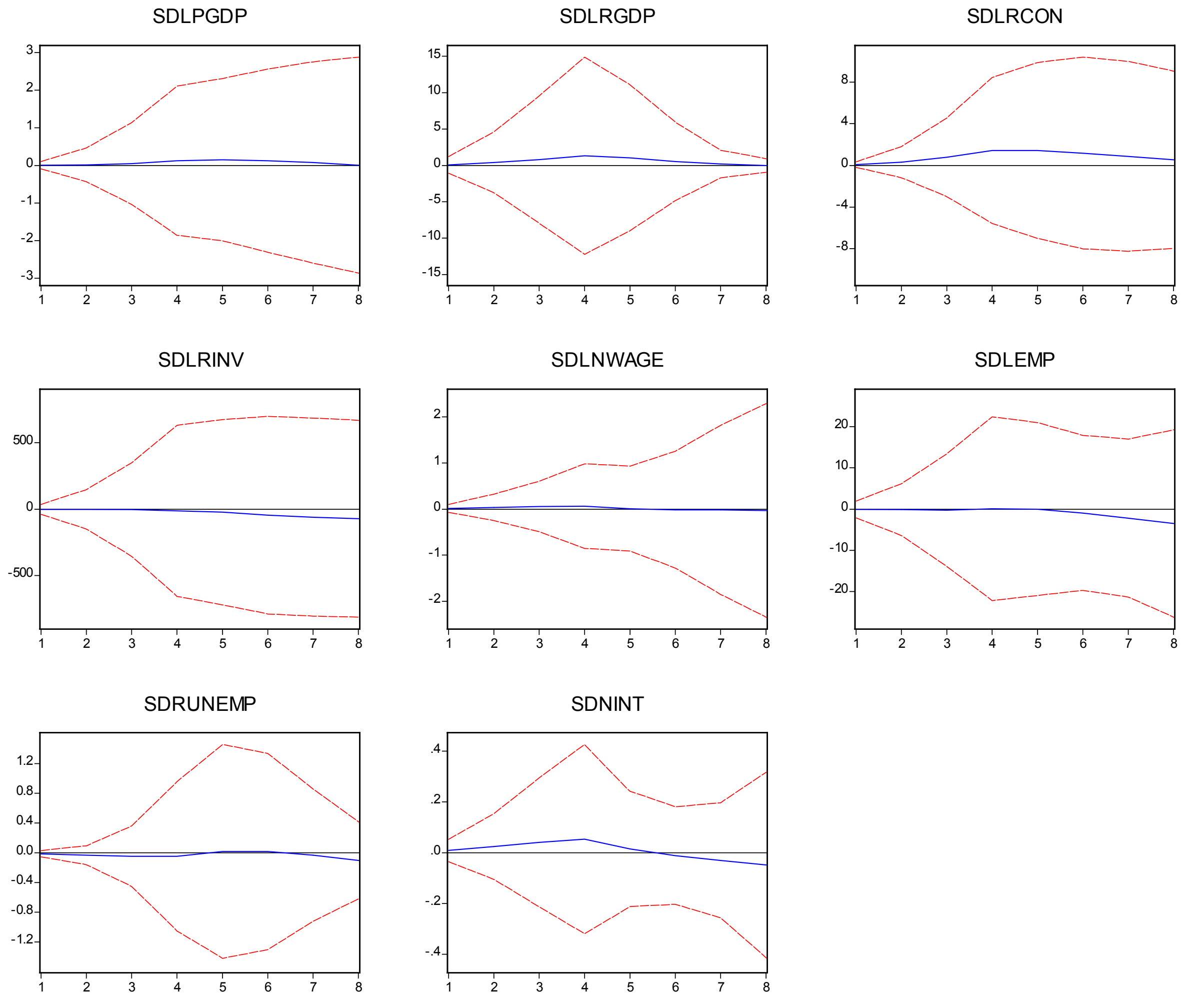
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 10. Mean squared prediction error differentials for ordinary differences



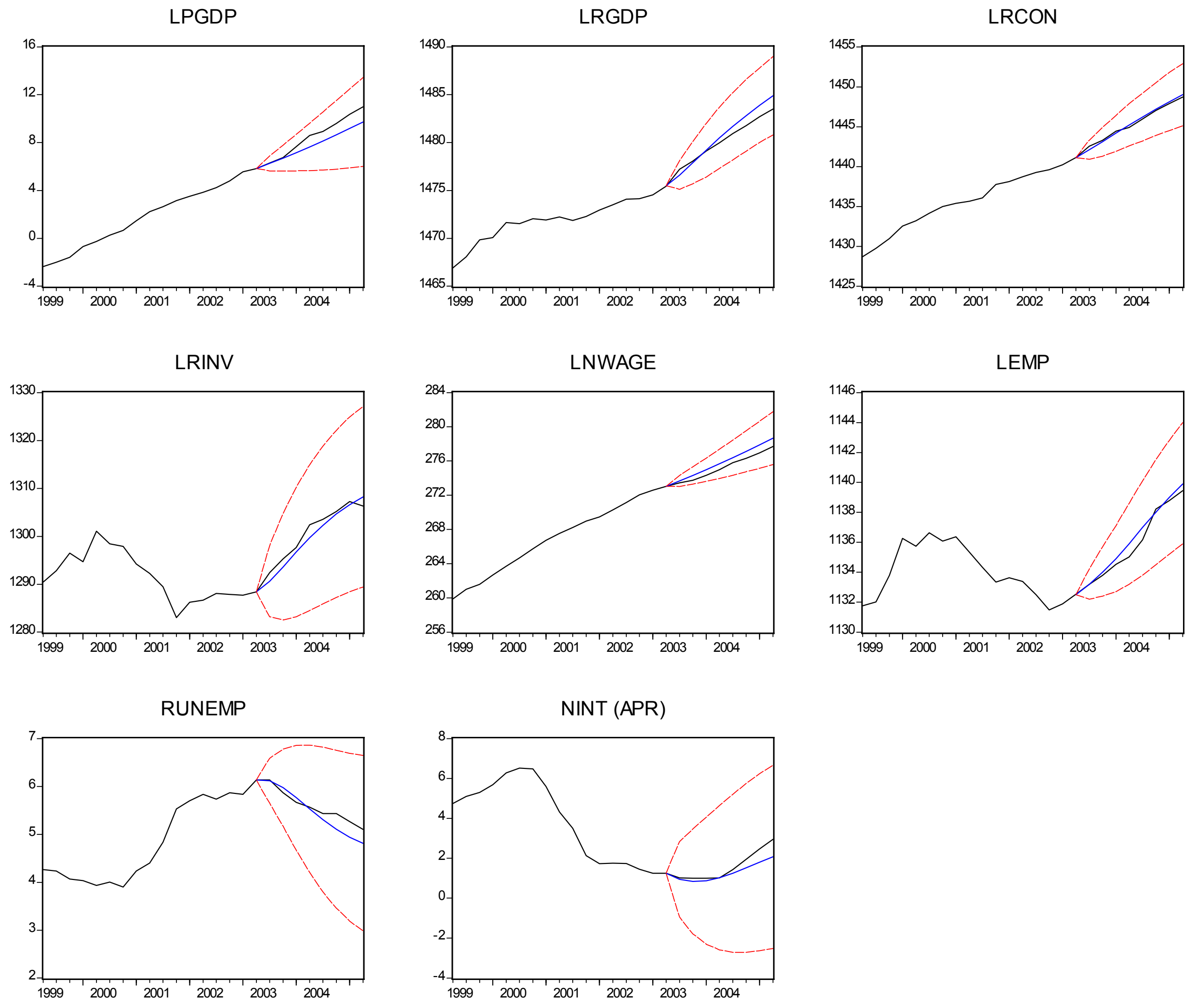
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 11. Mean squared prediction error differentials for seasonal differences



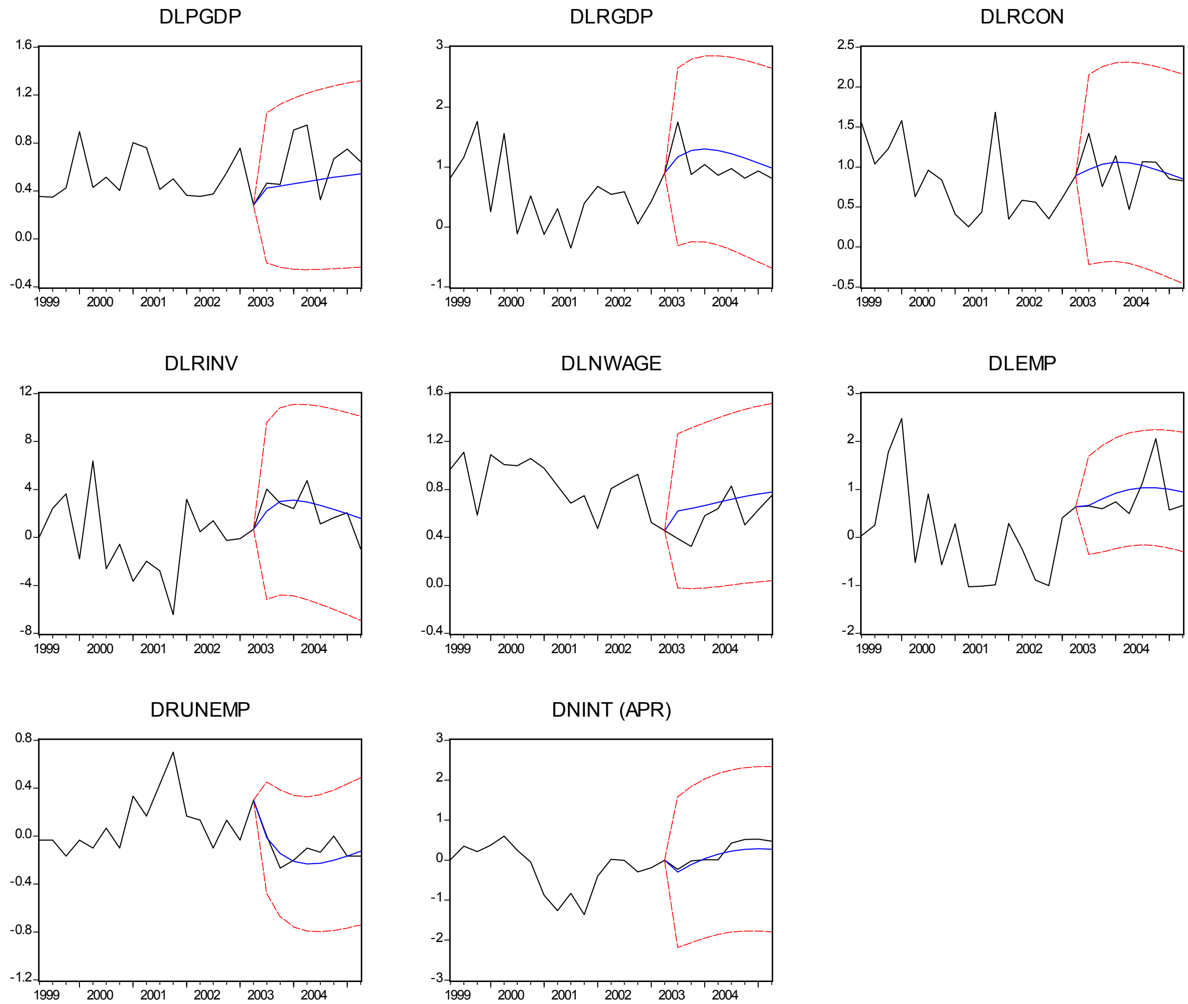
Note: Mean squared prediction error differentials are defined as the mean squared prediction error for the unobserved components model less that for the ARIMA model. Symmetric 95% confidence intervals account for contemporaneous and serial correlation of forecast errors.

Figure 12. Dynamic forecasts of levels of observed nonpredetermined endogenous variables



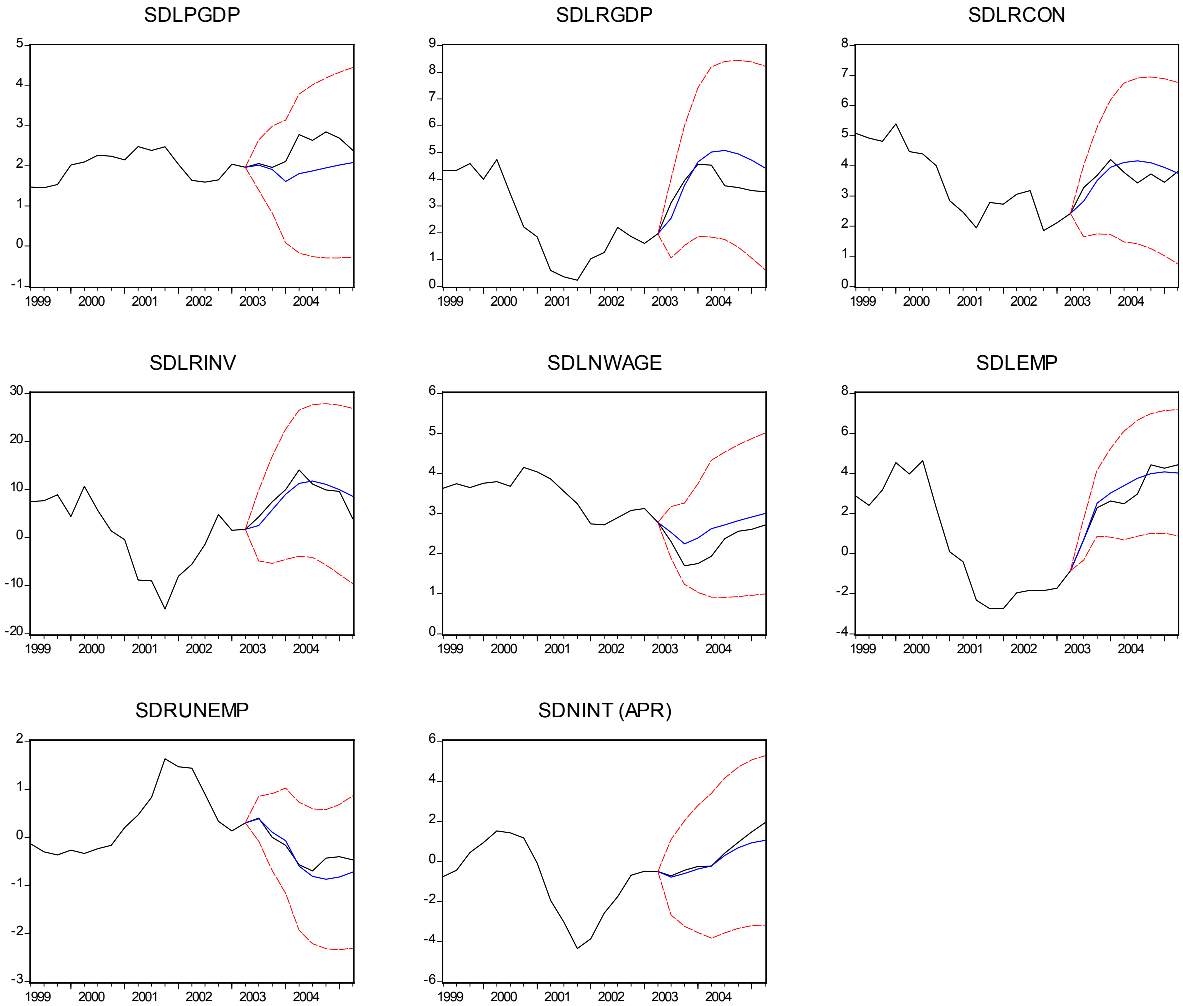
Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

Figure 13. Dynamic forecasts of ordinary differences of observed nonpredetermined endogenous variables



Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

Figure 14. Dynamic forecasts of seasonal differences of observed nonpredetermined endogenous variables



Note: Realized outcomes are represented by black lines, while blue lines depict point forecasts. Symmetric 95% confidence intervals assume multivariate normally distributed signal and state innovation vectors and known parameters.

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